[Alexander and Sadiku, 2009, Q11.12]

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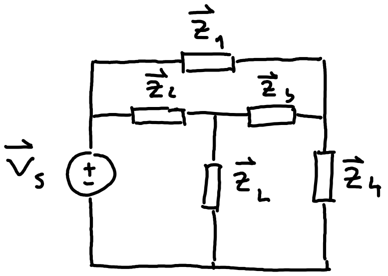



$\vec{Z}_1 = 2j\ \Omega$

$\vec{Z}_2 = 4\ \Omega$

$\vec{Z}_3 = -3j\ \Omega$

$\vec{Z}_4 = 5\ \Omega$

$\vec{V}_s = 40\ V$

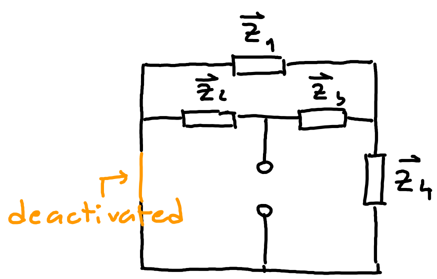
We want to find the value of $\vec{Z}_L$ so that maximum amount of power is transferred to it.

In class, we have seen that this kind of problem can be solve by first finding the thevenin equivalent of the rest of the circuit. Then, the optimal value of $\vec{Z}_L$ is $\vec{Z}_{th}^*$ and the corresponding maximum power is

$$P_{max} = \frac{|\vec{V}_{th}|^2}{8\underbrace{R_{th}}_{\mathrm{Re}\{\vec{Z}_{th}\}}}$$

Step (1) Find the thevenin equivalent circuit:

Find $\vec{Z}_{th}$

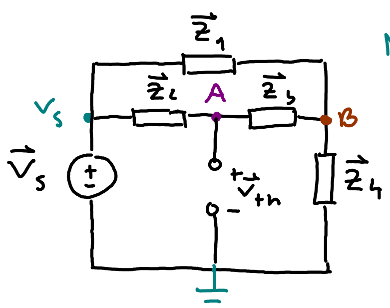



It might be difficult to see that $Z_1 /\!/ Z_4$ but they are.

Once you have this, then it is easy to get

$$\vec{Z}_{th} = \vec{Z}_2 /\!/ (\vec{Z}_3 + \vec{Z}_1 /\!/ \vec{Z}_4) = 0.823 - 0.864j\ \Omega$$

Next, we need to calculate $\vec{V}_{th}$. So, we turn $\vec{V}_s$ back on but still leave the two terminals at the load open (to find the open-circuit voltage).




Nodal analysis : Applying KCL

At A, we have

$$\frac{\vec{V}_A - \vec{V}_s}{\vec{Z}_2} + \frac{\vec{V}_A - \vec{V}_B}{\vec{Z}_3} = 0$$

$$\left(\frac{1}{\vec{Z}_2} + \frac{1}{\vec{Z}_3}\right)\vec{V}_A - \frac{1}{\vec{Z}_3}\vec{V}_B = \frac{\vec{V}_s}{\vec{Z}_2}$$

At B, we have

$$\frac{\vec{V}_B - \vec{V}_A}{\vec{Z}_3} + \frac{\vec{V}_B}{\vec{Z}_4} + \frac{\vec{V}_B - \vec{V}_s}{\vec{Z}_1} = 0$$

$$-\frac{1}{\vec{Z}_3}\vec{V}_A + \left(\frac{1}{\vec{Z}_1} + \frac{1}{\vec{Z}_3} + \frac{1}{\vec{Z}_4}\right)\vec{V}_B = \frac{\vec{V}_s}{\vec{Z}_1}$$

Cramer's rule

$$\vec{V}_A = \frac{\begin{vmatrix} \frac{\vec{V}_s}{\vec{Z}_2} & -\frac{1}{\vec{Z}_3} \\ \frac{\vec{V}_s}{\vec{Z}_1} & \frac{1}{\vec{Z}_1} + \frac{1}{\vec{Z}_3} + \frac{1}{\vec{Z}_4} \end{vmatrix}}{\begin{vmatrix} \frac{1}{\vec{Z}_2} + \frac{1}{\vec{Z}_3} & -\frac{1}{\vec{Z}_3} \\ -\frac{1}{\vec{Z}_3} & \frac{1}{\vec{Z}_1} + \frac{1}{\vec{Z}_3} + \frac{1}{\vec{Z}_4} \end{vmatrix}} = 38.6 - 12.146j = 40.46 \angle -17.47^\circ = \vec{V}_{th}$$

Step (2) For maximum power transfer, we need to use $Z_L = Z_{th}^*$

Therefore, $\vec{Z}_L = 0.823 + 0.864j \ \Omega$

The max power absorbed by this optimal $\vec{Z}_L$ is given by $\dfrac{|\vec{V}_{th}|^2}{8R_L} = 248.58\ W$

# [F2010] ac power analysis: max power transfer

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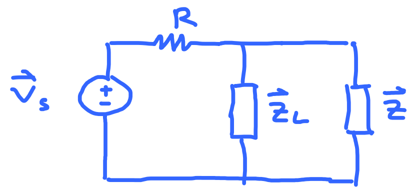
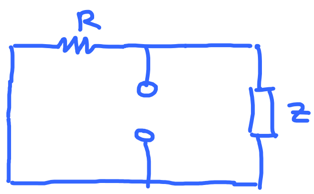
$\vec{V}_s = 7\angle 30^\circ$

$R = 6\Omega$

$\vec{Z} = j\omega L = j \times 200 \times 0.2 = 40j\ \Omega$




(a) We first find $\vec{Z}_{th}$ at the load terminals:

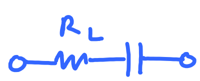



$\vec{Z}_{th} = \vec{Z} /\!/ R = 6 /\!/ 40j \approx 5.868 + 0.88j$

Optimal load impedance: $\vec{Z}_L = \vec{Z}_{th}^* \approx 5.868 - 0.88j$

(b) Note that the optimal $\vec{Z}_L$ that we got in part (a) has negative reactance. Therefore, we may construct this $\vec{Z}_L$ from


(There are many solutions to this problem.)

Method 1: Let's try 


We set $R = 5.868\ \Omega$.

With this construction, we have

$$\vec{Z}_L = 5.868 + \underbrace{\frac{1}{j\omega C}}_{\frac{-j}{\omega C}}.$$

So, we need $\frac{1}{\omega C} = 0.88 \Rightarrow C = \frac{1}{\underset{\underset{200}{\uparrow}}{\omega} \times 0.88} \approx 5.68\ mF$.

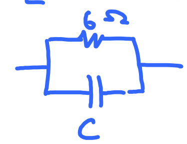
In conclusion, $\vec{Z}_L$ can be built by 


Method 2: Recall that $\vec{Z}_{th} = 6 /\!/ 40j = \frac{6 \times (40j)}{6 + 40j}$

and $\vec{Z}_L = Z_{th}^{*} = \dfrac{6\times(-40j)}{6+(-40j)}$

Writing this way, we see that $Z_L = 6 \,//\, (-40j)$

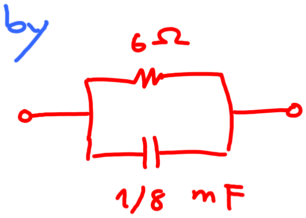
Therefore, we can construct $\vec{Z}_L$ by




where $\dfrac{1}{j\omega C} = -40j$.

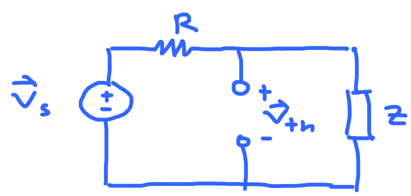
This gives $C = \dfrac{1}{40\omega} = \dfrac{1}{40\times 200} = \dfrac{1}{8000} = \dfrac{1}{8}$ mF.

In conclusion, $\vec{Z}_L$ can be built by




(c) Max power $= \dfrac{|\vec{V}_{th}|^2}{8R_{th}}$. So, we need to find $\vec{V}_{th}$ at the load terminals.

By the voltage divider formula,




$$\vec{V}_{th} = \frac{Z}{R+Z}\vec{V}_s .$$

So,

$$|\vec{V}_{th}|^2 = \frac{40^2}{6^2+40^2}\times 7^2 \approx 47.922 \text{ V.}$$

Max power $= \dfrac{|\vec{V}_{th}|^2}{8R_{th}} \approx \dfrac{47.922}{8\times 5.868} \approx$ 1.021 W

# [Alexander and Sadiku, 2012, Q11.26]

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We have shown in class that the effective value of a voltage or current is the same as its rms value.

Therefore, $V_{eff} = V_{rms} = \sqrt{\frac{1}{T_0}\int_0^{T_0} v^2(t)\,dt}$

for periodic voltage

Here, $T_0 = 4$. So, $V_{rms}^2 = \frac{1}{4}\left(\int_0^2 10^2\,dt + \int_2^4 20^2\,dt\right)$

$$= \frac{1}{4}\left(2\times 100 + 2\times 400\right) = \frac{1000}{4} = 250$$

$$V_{rms} = \sqrt{250} = 5\sqrt{10} \approx 15.8114 \text{ V}$$

An offset sinusoid $x(t)$ is a sinusoid of the form

$$x(t) = \underbrace{X_0}_{\text{This is called the "offset" or "dc offset".}} + \overbrace{X_m \cos(\omega t + \phi)}^{\text{This is the same old standard sinusoid that we have studied before.}}$$

Any sinusoid is periodic with period $T_0 = \frac{1}{f} = \frac{2\pi}{\omega}$.
Having offset does not change its periodicity nor its period.
So, offset sinusoid is still periodic with period $T_0$.

In which case, $X_{eff} = X_{rms} = \sqrt{\frac{1}{T_0}\int_0^{T_0} x^2(t)\,dt}$ .

This is the formula that we uses when $x(t)$ is periodic.

Pluggins in $x(t) = X_0 + X_m \cos(\omega t + \phi)$, we have

To avoid having to write "$\sqrt{\ }$" many times, we first find $X^2_{rms}$ instead of $X_{rms}$.

$$X_{rms}^{2} = \frac{1}{T_0}\int_0^{T_0} \left(X_0 + X_m \cos(\omega t + \phi)\right)^2 dt$$

$$= \frac{1}{T_0}\int_0^{T} \left(X_0^2 + 2X_0X_m \cos(\omega t + \phi) + X_m^2\cos^2(\omega t + \phi)\right) dt$$

$$= \underbrace{\frac{1}{T_0}\int_0^{T_0} X_0^2\,dt}_{= X_0^2} + \frac{1}{T_0}(2X_0X_m)\underbrace{\int_0^{T_0}\cos(\omega t + \phi)\,dt}_{\text{Integration over one period of sinusoid gives } 0.} + \underbrace{\frac{1}{T_0}\int_0^{T_0}\left(X_m \cos(\omega t + \phi)\right)^2 dt}_{\text{In class, we know that this part is } \left(\frac{X_m}{\sqrt{2}}\right)^2}$$

$$= X_0^2 + \frac{X_m^2}{2}$$

Therefore, $X_{rms} = \sqrt{X_0^2 + \frac{X_m^2}{2}}$

$i(t) = 10\ A \Rightarrow I_{rms} = \sqrt{10^2 + \frac{0^2}{2}} = 10\ A$

$v(t) = 4 + 3\cos 5t \Rightarrow V_{rms} = \sqrt{4^2 + \frac{3^2}{2}} = \sqrt{16 + \frac{9}{2}} = \sqrt{\frac{41}{2}} \approx 4.5277\ V$

$$v(t) = 4 + 3\cos 5t \Rightarrow V_{rms} = \sqrt{4^2 + \frac{3^2}{2}} = \sqrt{16 + \frac{9}{2}} = \sqrt{\frac{41}{2}} \approx 4.5277\ V$$

$$i(t) = 8 - 6\sin(2t)$$

$$= 8 + 6\cos(2t - 90^\circ + 180^\circ)$$

$$= 8 + 6\cos(2t + 90^\circ) \Rightarrow I_{rms} = \sqrt{8^2 + \frac{6^2}{2}} = \sqrt{64 + 18} = \sqrt{82} \approx 9.0554\ V$$

# Solution of the first order ODE

Tuesday, September 17, 2013 9:17 PM

Recall formula (F1) that we discussed in class:

When $\frac{d}{dt}x(t) = a\,x(t) + b$,

with initial condition $x(t_0) = x_0$,

Then,

$$x(t) = e^{a(t-t_0)}\left(x_0 + \frac{b}{a}\right) - \frac{b}{a}.$$

(a) $\frac{d}{dt}x(t) + 5x(t) = 0 \Rightarrow \frac{d}{dt}x(t) = \underbrace{-5}_{a=-5}x(t)\ \underbrace{\ }_{b=0}$

$x(0) = 5 \Rightarrow t_0 = 0$ and $x_0 = 5$.

Therefore,

$$x(t) = e^{-5(t-0)}\left(5 + \frac{0}{-5}\right) - \frac{0}{-5}$$

$$= 5e^{-5t}.$$

Check: $x(0) = 5e^{-5\times 0} = 5$ ✓

$\frac{d}{dt}x(t) = \underbrace{5e^{-5t}}_{x(t)}(-5) = -5x(t) \Rightarrow \frac{d}{dt}x(t) + 5x(t) = 0$ ✓

(b) $\frac{d}{dt}x(t) = \underbrace{-3}_{a=-3}x(t) + \underbrace{2}_{b=2}$, $\quad x(0) = 0. \quad t_0 = 0 \quad x_0 = 0$

$$\Rightarrow x(t) = e^{-3(t-0)}\left(0 + \frac{2}{-3}\right) - \frac{2}{-3}$$

$$= -\frac{2}{3}e^{-3t} + \frac{2}{3}$$

Check: $x(0) = -\frac{2}{3}e^{-3\times 0} + \frac{2}{3} = -\frac{2}{3} + \frac{2}{3} = 0$ ✓

$\frac{d}{dt}x(t) = -\frac{2}{3}e^{-3t}(-3) = -3x(t) - \frac{2}{3}(-3) = -3x(t) + 2$ ✓

$$= x(t) - \frac{2}{3}$$

[Alexander and Sadiku, 2012, Q7.8]

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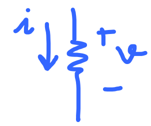
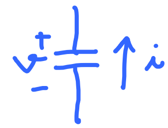
(a) By Ohm's law, $v = iR$.




Therefore, $R = \frac{v}{i} = \frac{10e^{-4t}}{0.2e^{-4t}} = \frac{10}{0.2} = 50\,\Omega$

Note also that, for the capacitor, we have




$$i = -C\frac{dv}{dt}.$$

Note that we have an extra minus sign here because $v$ and $i$ do not conform with the passive sign convention.

Plugging in the provided expressions of $i$ and $v$, we have

$$0.2\cancel{e^{-4t}} = -C \times 10\cancel{e^{-4t}} \times (-4)$$

$$\Rightarrow \quad C = \frac{0.\cancel{2}}{10\times\cancel{4}_{2}} = \frac{1}{200} = 5\text{ mF}$$

(b) $\tau = RC = 50 \times 5\text{m} = 0.25$ s.

Remark: Alternatively, we can also get the value of $\tau$ directly from the expression of $v(t)$.

Review: For source-free RC circuit, we have

$$v_c(t) = V_0 e^{-(t-t_0)/\tau}.$$

This could be rewritten as

$$v_c(t) = \underbrace{V_0 e^{t_0/\tau}}\ \underbrace{e^{-t/\tau}}.$$

This part is time-dependent.

This part is simply some constant.

It is given that $v_c(t) = 10e^{-4t}$.

By comparison with the general expression above, we know that $\frac{1}{\tau} = 4$

Therefore, $\tau = \frac{1}{4} = 0.25$ s ← Same answer as what we found above.

We can then find R using Ohm's law as shown above.

Then, $C \underset{\tau = RC}{\underset{\uparrow}{=}} \tau / R = \frac{1}{4} \Big/ 50 = \frac{1}{200}$ ← Same answer as what we found above.

(c) To calculate the energy in the capacitor, we use $w_c(t) = \frac{1}{2} C v_c^2(t)$.

Here, we want the initial energy. So, we find

$$w_c(0) = \frac{1}{2} C (v(0))^2.$$

Now,

$$v(0) = v(0^+) = 10 e^{-4 \times 0} = 10 \text{ V}$$

no voltage jump across capacitor

Therefore,

$$w_c(0) = \frac{1}{2} \times 5 \times 10^{-3} \times 10^2 = 0.25 \text{ J}$$

(d) $w_c(t) = \frac{1}{2} C \times v^2(t)$

$w_c(0) = \frac{1}{2} C v^2(0)$

Want to find t such that

$$w_c(t) = \frac{1}{2} w(0)$$

$$\cancel{\tfrac{1}{2}} \cancel{C} v_c^2(t) = \cancel{\tfrac{1}{2}} \times \frac{1}{2} \cancel{C} v^2(0)$$

$$v_c(t) = \frac{1}{\sqrt{2}} v_c(0)$$

$$\cancel{10} e^{-4t} = \frac{1}{\sqrt{2}} \cancel{10}$$

$$t = \frac{\ln \sqrt{2}}{4} = 87 \text{ ms}$$

# [Alexander and Sadiku, 2012, Q7.3]

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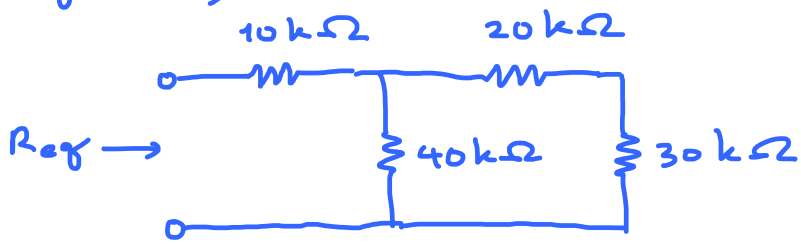
For RC circuit, the time constant is given by $\tau = RC$.

When there are >1 resistors, we first find the equivalent resistance $R_{eq}$ at the terminals of the capacitor.

In this question,




$R_{eq}$ at the terminals of capacitor

$$= 10 + 40 // (20+30) \ k\Omega$$

$$= 10 + 40 // 50 = 290/9 \ k\Omega$$

$$\tau = R_{eq} \times C = \frac{290}{9} k \times 100 \times 10^{-12} = 3.22 \mu s$$